Stirling's Approximation:-

For factorial of very large numbers (such as Avogadro's number) Stirling's approximation is a useful tool for mathematical manipulation.

How to approximate $N!$ for large number?

Asymptotic approximation → Approximation to a function which improves as the argument of that function increases.

Since $N!$ involves product of numbers, therefore, it is convenient to deal with $\ln N!$ (which is sum of numbers.)

⇒ The asymptotic approximation to $\ln N!$ is called Stirling approximation.

Now ~~$N! = N(N-1)(N+2)$~~

$$N! = N(N-1)(N-2)\cdots(3)(2)(1) \quad \text{——— (1)}$$

$$\ln N! = \ln N + \ln N-1 + \ln N-2 + \cdots + \ln 3 + \ln 2 + \ln 1$$

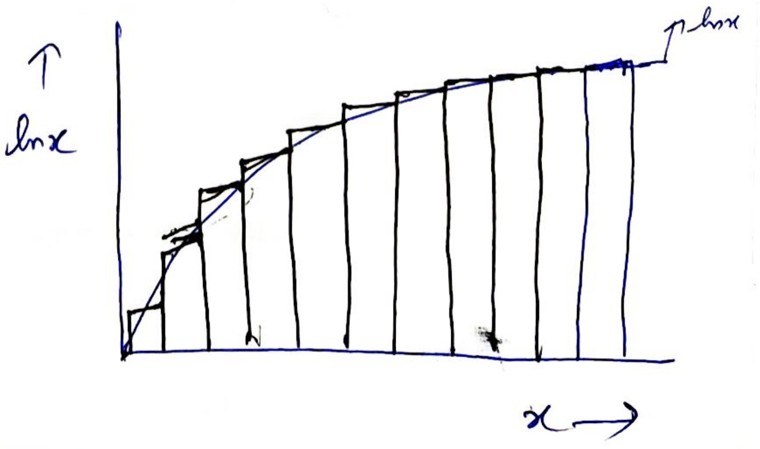
or $$\ln N! = \sum_{m=1}^{N} \ln m \quad \text{——— (2)}$$

See figure →

The sum of areas under rectangles upto N is $\ln N!$

Fig also shows continuous curve $\ln x$

→ $\ln x$ forms an envelope to rectangles.



which is smoother approximations to rectangles as x increases

In the beginning approximations is poor.

But for large N, we can approximate $\ln N!$ as

$$\ln N! = \sum_{m=1}^{N} \ln m \approx \int_{1}^{N} \ln x \, dx = N\ln N - N$$

or $\boxed{\ln N! = N\ln N - N}$ → Stirling approximation